



IB MYP YEAR 5

YEAR 10 Extended
Mathematics

Assessment #2
TRANSFORMATION OF FUNCTIONS

Name: ANSWERS (10)

Teacher: Ms. Li, Mr. So & Mr. Wong

Date of task: Friday, November 2, 2012

Time allowed: 60 minutes

Student's Performance in Different Criterion			
B		C	

INSTRUCTIONS:

- ◆ Read the instructions for all questions carefully.
- ◆ Show all work, steps and proper units.
- ◆ Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- ◆ Write in **PENCIL**.
- ◆ **GDC** is allowed.
- ◆ Allowed to use **non-electronic dictionary**.

ASSESSMENT:

- ◆ Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- ◆ This task assesses Criteria **B & C** considering ALL the questions.
 - ✧ Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion B: INVESTIGATING PATTERNS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1–2 Do Maths	<ul style="list-style-type: none"> ● The student has answered the early questions in Part A, generally correctly. ● The method(s) used are generally appropriate. 	The student applies, with some guidance , mathematical problem-solving techniques to recognize simple patterns.	
3–4 General Rule	<ul style="list-style-type: none"> ● The student has been successful with most of the questions asked in Part A, and perhaps some of the easier questions in Part B (Q8 and Q9) ● The student has been generally successful with all questions asked in Part A, and many of the questions in Part B. Answers are accurate, and method is clear. 	<p>The student</p> <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, and ● suggests relationships or general rules. 	Teacher's Final Grade
5–6 Test it	<ul style="list-style-type: none"> ● The student has been generally successful with all questions asked in Part A, and many of the questions in Part B, including the unfamiliar ones where the mirror line is not an axis. ● Answers are accurate, and method is clear. 	<p>The student</p> <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, ● describes them as relationships or general rules, and ● draws conclusions consistent with findings. 	(0-8)
7–8 Prove it	<ul style="list-style-type: none"> ● The student has answered the early questions in Part A, generally correctly. ● The method(s) used are generally appropriate. 	<p>The student</p> <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, ● describes them as relationships or general rules, ● draws conclusions consistent with findings, and ● provides justifications or proofs. 	

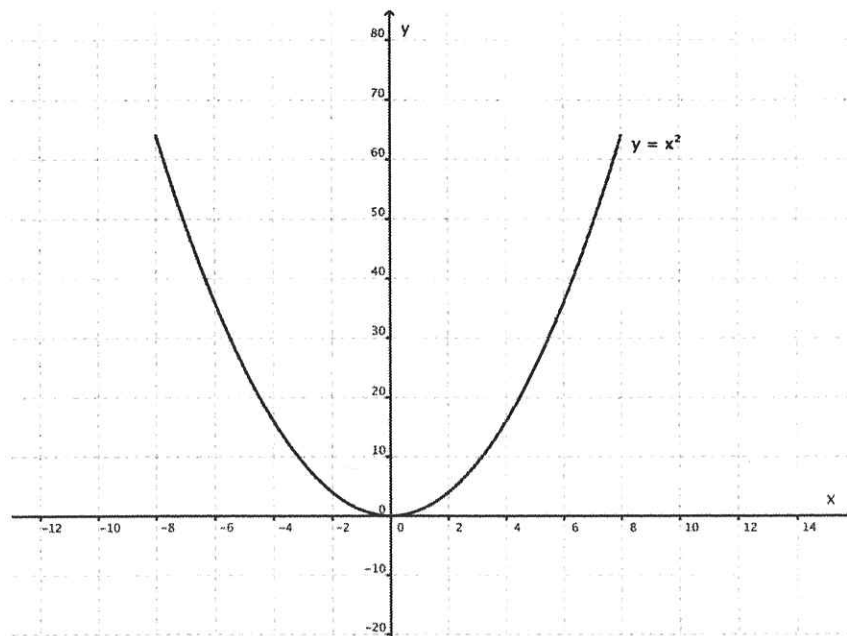
Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-6)
1-2	There are some errors or inconsistencies in use of terminology. There are some errors in the writing of equations. Narrative is difficult to follow.	<ul style="list-style-type: none"> The student shows basic use of mathematical language and/or forms of mathematical representation. The lines of reasoning are difficult to follow. 	
3-4	Generally students use correct terminology accurately, with only a few errors. Equations are mostly written clearly and accurately. Narrative can be followed, and diagrams are clear and labeled.	<ul style="list-style-type: none"> The student shows sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning are clear though not always logical or complete. The student moves between different forms of representation with some success. 	Teacher's Final Grade
5-6	There are some errors or inconsistencies in use of terminology. There are some errors in the writing of equations. Narrative is difficult to follow.	<ul style="list-style-type: none"> The student shows good use of mathematical language and forms of mathematical representation. The lines of reasoning are concise, logical and complete. The student moves effectively between different forms of representation. 	(0-6)

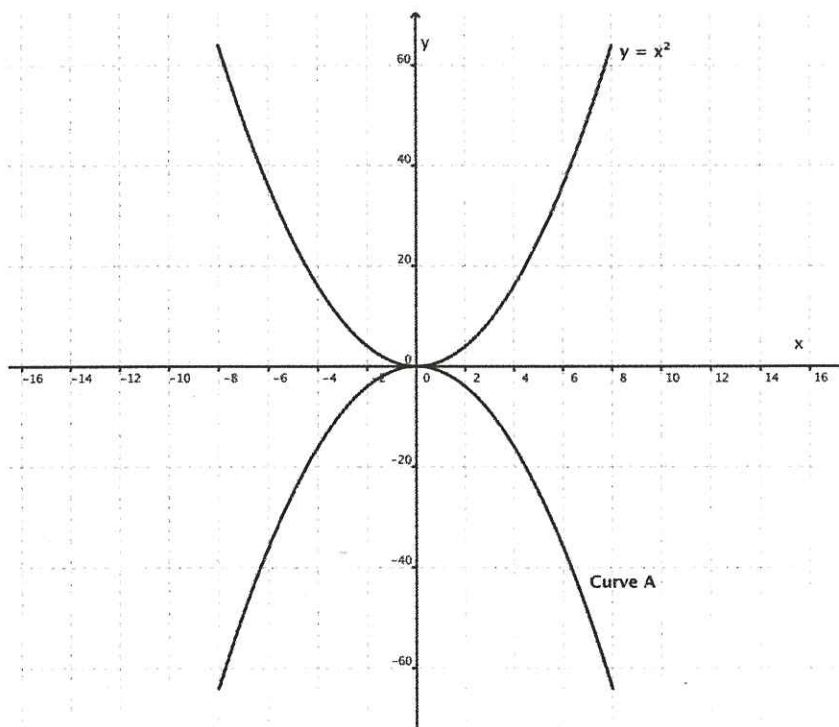
PART A (Suggested time: 20 minutes)

When the designers of the Hong Kong MTR were creating the now-famous logo, they decided to use transformations of functions.

They started with a parabola as in the graph below:



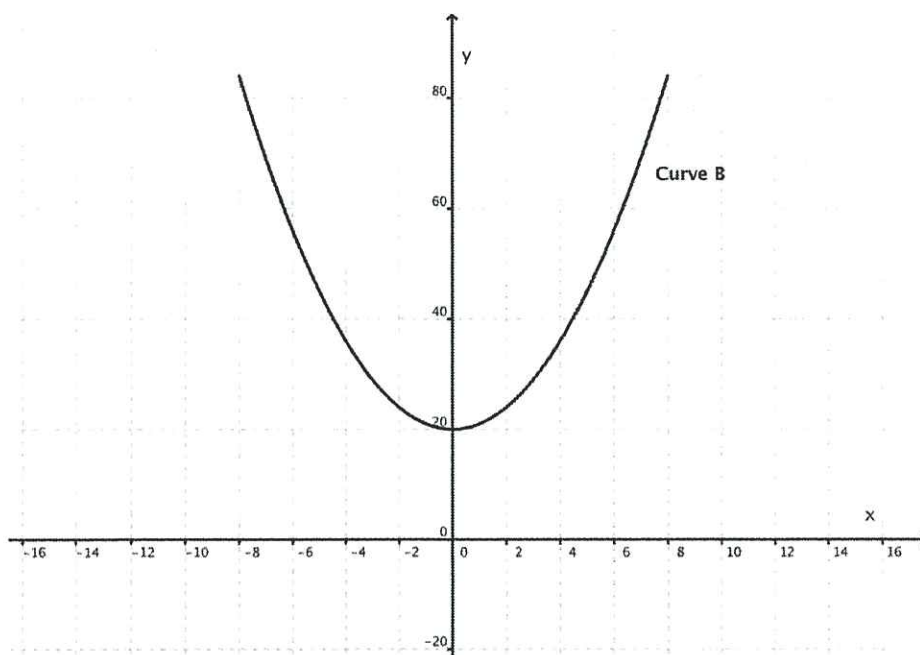
They then reflected this curve by the x-axis:



1. If the **starting curve** has the equation $y = x^2$, what is the **equation of curve A**?

$$y = -x^2$$

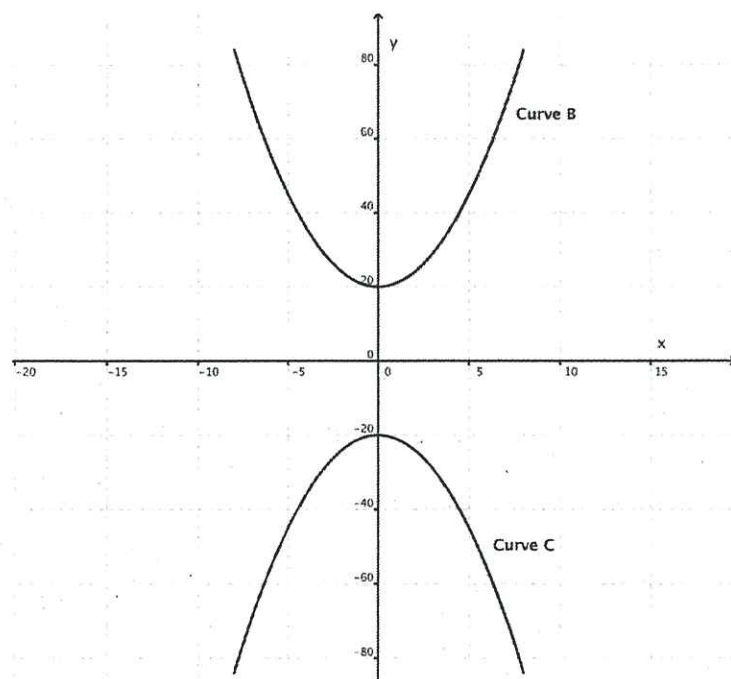
The designers then performed a transformation from their **starting curve**. It became this:



2. What do you think the **equation of curve B** is?

$$y = x^2 + 20$$

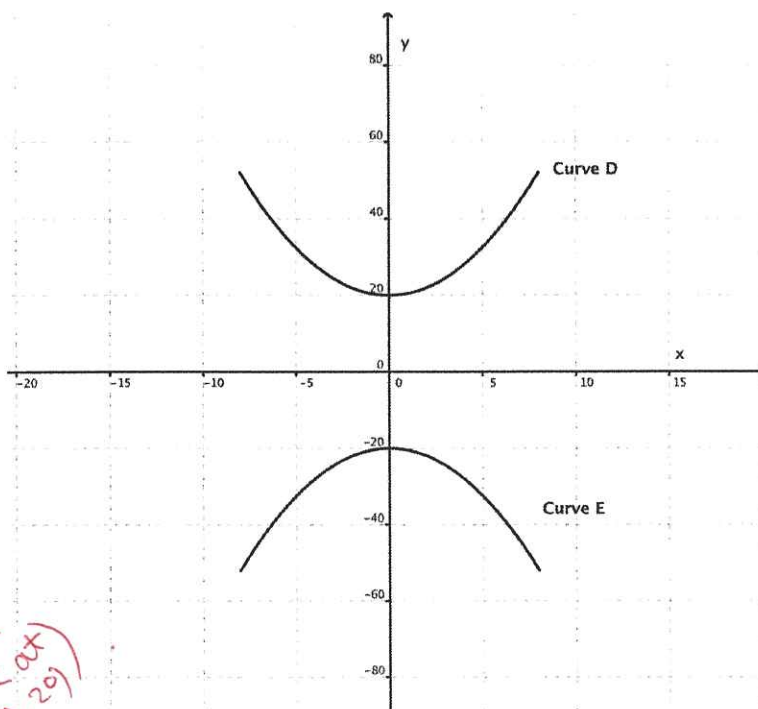
After the designers **reflected curve B** by the **x-axis**, they got **curve C**:



3. What do you think the **equation of curve C** is?

$$y = -x^2 - 20$$

The designers then went on to make one further alteration. The diagram below shows the results of that alteration. (Curves D and E are sketched on the same axes as B and C were on the last graph):



explain why not contracted in y direction (look at (0, 20))

4. Describe in words what this last alteration was (to make **curve B become curve D** and **C become E**)?

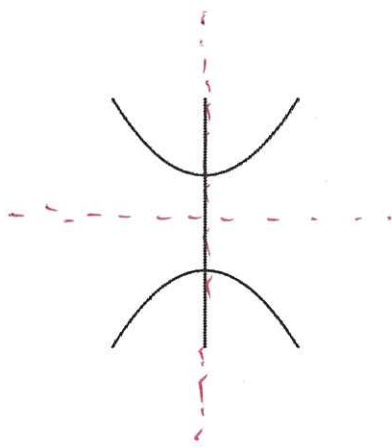
stretch by factor of $a=\sqrt{2}$ in x direction

5. What do you think the **equation of curve D** is?

$$y = \left(\frac{1}{\sqrt{2}}x\right)^2 + 20 = \frac{1}{2}x^2 + 20$$

6. What do you think the **equation of curve E** is?

$$y = -\frac{1}{2}x^2 - 20$$



7.

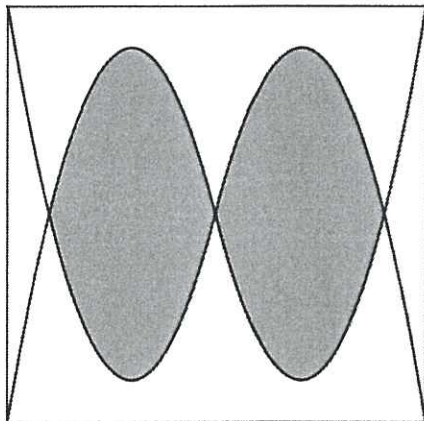
The "finished" logo is on the left. Describe fully the symmetry of the design

2 lines of symmetry ($y=0$ $x=0$
 x -axis, y -axis)

rotational symmetry of 2.

PART B (Suggested time: 40 minutes)

Here is a logo of a well-known sportswear company:

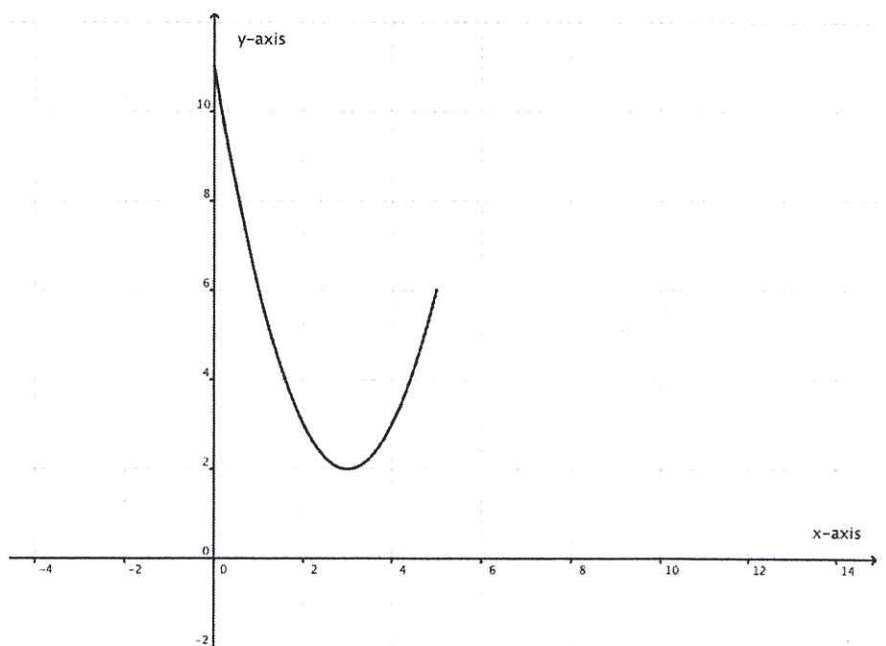


Two students, Ahmed and Delinda, are interested in the mathematics of the curves that make up the logo, and the relationship between the curves.

Ahmed believes that the design is really based on a simple curve (a quadratic), which is then transformed using simple transformations.

He says that the "basic unit" of the design below is

$$y = x^2 - 6x + 11 \quad 0 \leq x \leq 5$$



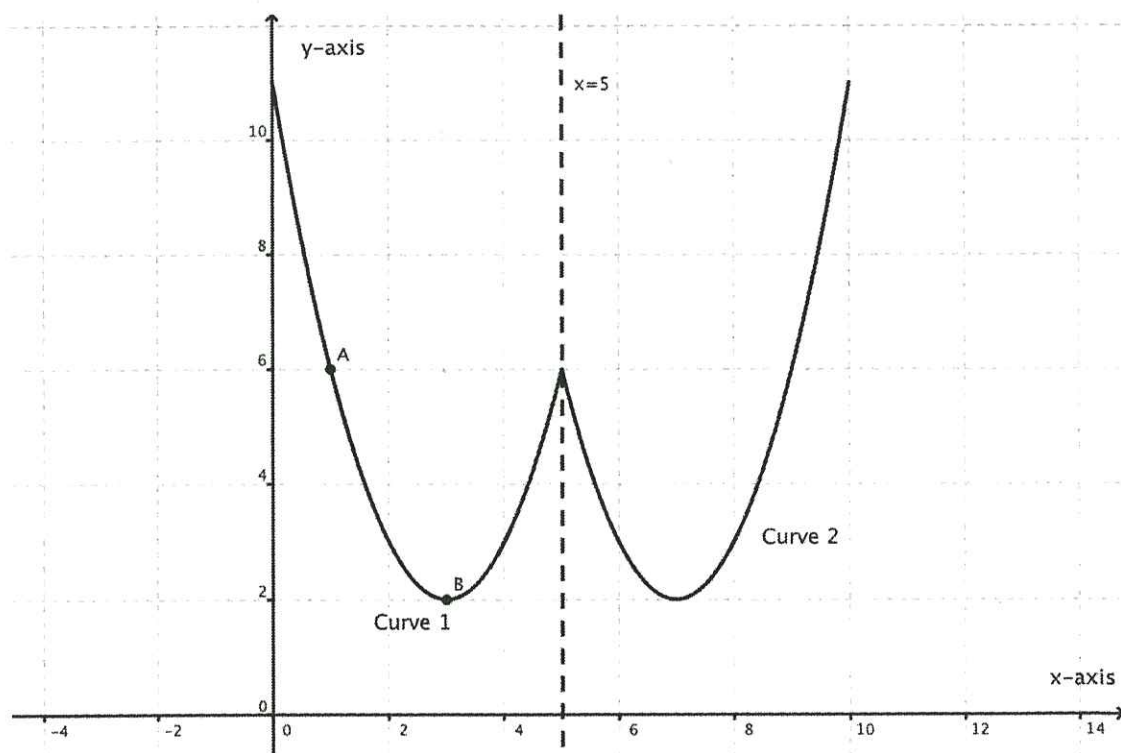
8. Show that the equation $y = x^2 - 6x + 11$ can be written as the **second form** $y = (x - 3)^2 + 2$.

$$y = (x - 3)^2 - 9 + 11$$
$$y = (x - 3)^2 + 2$$

9. Describe how this second form of the equation matches up with key features of the graph?

vertex / minimum = $(3, 2)$ ^{h, k}

He goes on to say that this curve is **reflected in the line $x = 5$** as shown below:



10. A is the point $(1, 6)$. What are the coordinates of the image of A after reflection in $x = 5$?

$(9, 6)$

11. B is the point (3, 2). What are the coordinates of the image of B after reflection in $x = 5$?

(7, 2)

12. P is the general point (x, y). What are the coordinates of the image of P after reflection in $x = 5$?

(10-x, y)

13. Ahmed takes the x-coordinate of his answer to Q12, and substitutes this in for x in the original equation $y = x^2 - 6x + 11$. He (correctly) believes that it gives him the equation of curve 2. What equation for curve 2 does Ahmed get, and what range of values of x does it apply to?

$$y = (10-x)^2 - 6(10-x) + 11$$

$$y = 100 - 20x + x^2 - 60 + 6x + 11$$

$$y = x^2 - 14x + 51$$

$$5 \leq x \leq 10$$

Delinda believes that Curve 1 in the diagram above can be transformed into Curve 2 **by a different way**.

14. What transformation(s) is/are Delinda thinking of? Give as many details as possible.

horizontal translation (4) 4 to right.

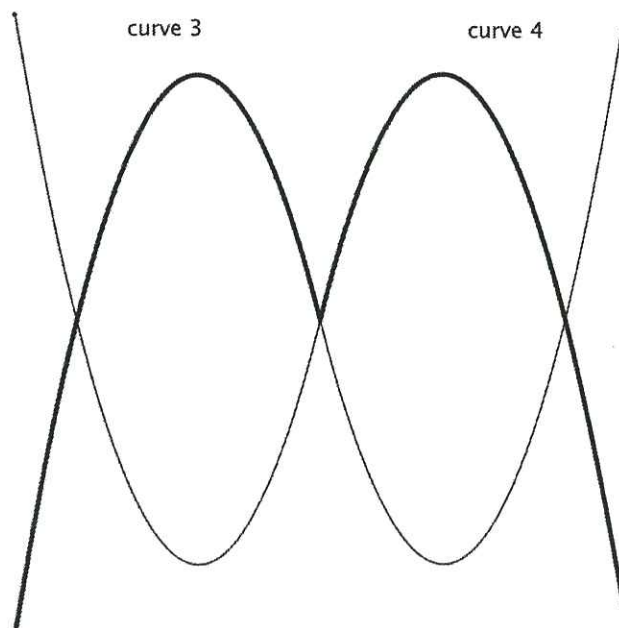
$$y = (x-4)^2 - 6(x-4) + 11$$

$$y = x^2 - 8x + 16 - 6x + 24 + 11$$

$$y = x^2 - 14x + 51$$

same

15. Is it possible for both Ahmed and Delinda to be right? Explain your answer.



To complete the above logo, curve 1 is transformed into curve 3, and curve 2 is transformed into curve 4

16. What single transformation is performed on curves 1 and 2 in order to end up with the logo?

reflection $y=6$

17. What is the equation of curve 3?

$$P(x, y) \rightarrow P'(x, 12-y) \rightarrow 2 \text{ transformations}$$

$$12-y = x^2 - 6x + 11$$

$$y = -x^2 + 6x + 1$$

in 2 transformations $y = x^2 - 6x + 11$

① reflect on x axis

$$y = -(x^2 - 6x + 11)$$

$$= -x^2 + 6x - 11$$

② translate up by 12 (vector $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$)

$$y = (-x^2 + 6x - 11) + 12$$

$$= -x^2 + 6x + 1$$

End of Assessment

$$= -(x^2 - 6x) + 1$$

$$= -((x-3)^2 - 9) + 1$$

$$y = -(x-3)^2 + 10$$